Exchange Rate Pass Through, Cost Channel to Monetary Policy Transmission, Adaptive Learning, and the Price Puzzle

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Abstract
Using a New Keynesian open economy model, where the supply side effects of the exchange rate pass through as well as the cost channel of monetary policy transmission are taken into account, this paper evaluates the possibility of the price puzzle, which refers to anomalous behavior of inflation to a monetary shock. Unlike the existing studies, we consider the possibility of the price puzzle when agent expectations are based on adaptive learning and three monetary policy alternatives (the optimal monetary policy, money growth targeting, and a Calvo-type policy rule) are available to the central bank. Furthermore, we use two alternative measures of inflation. Calibration of our medium scale model, using plausible parameter values, reveals that irrespective of how the inflation rate is measured and the policy rule used by the central bank, the puzzle fades away when a sufficiently strong exchange rate pass through is present in the economy. We also find that a decrease in inflation is associated with a cost to the society in the form of lower aggregate output but this loss is minimum when the central bank uses the optimal monetary policy.

JEL Classification
E31, E32, E52

KeyWords
Learning, Exchange rate pass through, price puzzle, cost channel, Calvo rule, optimal monetary policy

1. Introduction
Central Banks around the globe tend to use a high interest rate policy to control the rate of inflation. However, a number of empirical studies found that an increase in the US Federal

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Funds Rate increases (rather than decreasing) the inflation rate. This unexpected result is known as the price puzzle (Eichenbaum, 1992). Theoretically speaking, an increase in the nominal interest rate decreases the aggregate demand, which depresses the upward price momentum. However, two possibilities might plague the expected negative correlation between the interest rate and prices. First, an increase in the nominal interest rate may result in a decrease in the real interest rate, which increases the demand for goods and services thereby increasing the rate of inflation. The other possibility is that a cost channel of monetary policy transmission exists. If firms borrow money to finance the cost of production, the marginal cost of production, as indicated by Ravenna and Walsh (2006), among others, is affected by the interest rate. In such a case, a contractionary monetary policy shock increases the marginal cost of production, which contributes to an increase in the prices of goods and services, thereby increasing the rate of inflation. Within the context of the price puzzle, a number of studies have examined the role of the cost channel. However, relatively few existing studies have focused on the role of exchange rate pass through, which is defined as the effect of exchange rate changes on the rate of inflation.

Studies that highlight the role of exchange rate pass through in real economies include Devereux and Engel (2003), Benigno and Benigno (2003), Sutherland (2005a & 2005b). These studies show that, in the case of an incomplete exchange rate pass through, the exchange rate volatility can have a direct impact on consumer welfare and hence the optimal monetary policy must take into account the exchange rate volatility. A number of studies have examined the role of exchange rate pass through in the context of imported input prices."
studies (for example see Bachetta and van Wincoop, 2003; Gagnon and Ihrig, 2004; Campa and Goldberg, 2006; and Takhtamanova, 2010) focus on the relationship between exchange rate pass through and the aggregate price indices. Using a small scale dynamic stochastic general equilibrium (DSGE) model where agent expectations are formed rationally, the theoretical predictions of Ali and Anwar (2016) suggest that exchange rate pass through can play a vital role in determining the direction of the correlation between the interest rate and inflation.

Using a medium scale DSGE model, where three types of monetary policy alternatives are available and agent expectations are based on adaptive learning, we argue that a contractionary monetary policy that leads to exchange rate appreciation can put a downward pressure on domestic prices due to exchange rate pass through. At the same time, owing to the cost channel of monetary policy, the increase in the interest rate also increases the cost of production, which puts an upward pressure on domestic prices. As the interest rate has both negative and positive effects on the cost of production, its net effect on cost push inflation is ambiguous. Inflation may increase, if the interest rate effect dominates the exchange rate effect and vice-versa. We consider two alternative measures of inflation: (i) the consumer price index based inflation rate and (ii) the general price index (which includes only domestically produced goods) based inflation rate. We refer to the second measure as the domestic inflation rate. Another distinguishing feature of this paper is that, we consider three alternative monetary policy rules: (i) the optimal monetary policy, (ii) money growth targeting, and (iii) a Calvo-type interest rate rule. This allows us to establish the robustness of our main result.

In this paper, we do not rely on the assumption of rational expectations, which is based on the idea that agents know the structure of the economy as well as the values of some key economic model parameters. Evans and Honkapohja (2001), among others, argue that even the
trained economists do not have prior knowledge of the values of economic model parameters. Economists tend to use historical data to estimate the relevant parameters. Milani (2007), among others, argues that DSGE models based on the assumption of rational expectations cannot mimic the persistence of aggregate output and inflation in particular. Millani showed that DSGE models based on adaptive learning can outperform the models based on rational expectations. Prior to Millani’s study, Bullard and Eusepi (2005), Cogley and Sargent (2005), among others, used the idea of adaptive learning to examine the post-war evolution of the US inflation and monetary policy. They concluded that models based on learning are helpful in understanding certain historical episodes that were harder to explain by means of model that are based on the idea of rational expectations. In summary, adaptive learning has emerged as an alternative to the idea of rational expectations. In the case of adaptive learning, economic agents are assumed to making use of an econometric model to forecast the future values of economic variables. As the new information becomes available, agents revise their expectations formation (or forecasting) rules. Agents learn through their experiences concerning the seriousness of the central bank in controlling the rate of inflation and go along with the plan of the central bank. In short, in the case of adaptive learning, the job of the central bank becomes much easier as there are no misgivings on the part of economic agents.

Due to the assumption of adaptive learning, it is not possible to derive analytical results and hence, using plausible parameter values, we resort to model calibration. Calibration of our medium scale DSGE model confirms that the cost channel of monetary policy is a necessary condition for price puzzle to occur under all three monetary policy alternatives. Furthermore, we find that some degree of exchange rate pass through is necessary to resolve the puzzle. Another very interesting finding of our model is that, in the presence of a moderate degree of exchange
rate pass through, the consumer price index (CPI) based inflation (which takes fluctuations in the prices of the imported goods and services into account) falls but domestic inflation (which excludes the direct effect of the exchange rate on inflation and which is measured by a general price index) continues to increase under all three monetary policy alternatives. However, in the presence of a strong exchange rate pass through, even the domestic inflation falls.

The rest of this paper is organized as follows. A brief literature review is provided in Section 2. Section 3 contains a description of our New Keynesian open economy model. Simulation results are presented and discussed in Section 4. Section 5 concludes the paper.

2. Related Literature

The literature on price puzzle, which is an empirical phenomenon, can be divided into a few categories as follows:

2.1 VAR/SVAR-based studies

Using a vector autoregressive (VAR) approach, Sims (1992) found that an increase in the interest rate leads to increase in inflation in several industrialized economies. Sims found that the extent of the price puzzle decreases, if we use the index of goods prices as a proxy for inflation. Sims suggested that the central bank should consider using an ‘information variable’ to determine the rate of inflation and then react preemptively.5 Hanson (2004), while using Bernanke and Mihov (1998) model to re-examine the price puzzle, reported that over the years the extent of the price puzzle has decreased but it continues to inhabit in monthly data. Using quarterly and mixed frequency data, Galles and Portier (2005) found that price puzzle was not very visible in the quarterly data. Using quarterly data from the US to estimate a VAR model,

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5 Prior to Sims (1992), using a theoretical model, Sargent and Wallace (1981) showed that when public debt is high and the real rate of return on government securities exceeds the growth rate of the economy, a tight monetary policy (in the form of a low growth rate of money supply) can result in higher inflation.
Castelnuovo and Surico (2005) found evidence of the price puzzle over the 1966 to 1979 period but there was no price puzzle over the 1979 to 2002 period. They attributed this result to differences in the conduct of the US monetary policy in the two sample periods. They argued that, up until 1979, the US monetary policy was not tight enough to control the rate of inflation. In such a situation, an increase in the nominal interest rate (as explained earlier) can result in a decrease in the real interest rate, which gives rise to the price puzzle. Using a DSGE model to generate the relevant data, Castelnuovo and Surico (2006) estimated a structural VAR (SVAR) model where monetary policy was passive. Their impulse response function predicted the presence of a price puzzle even in a model that excluded the cost channel to monetary policy. Irvendi and Guloglu (2010) used a structural vector error correction model (SVECM) to consider the issue of the price puzzle in five inflation targeting countries (i.e., Australia, New Zealand, Canada, Sweden, and the UK). They found that contractionary monetary policy shocks do not give rise to price puzzle in these countries.

2.2 Model Misspecification and the Price Puzzle

A number of existing studies have described the price puzzle as a model misspecification problem. For example, Castelnuovo (2007) showed that if the interest rate smoothing policy used by central banks was explicitly included in the model then price puzzle will not occur even in the presence of a cost channel. Likewise, Auray and Feve (2008) pointed out that price puzzle occurs when, despite price flexibility, monetary authorities choose a money supply rule over the interest rate rule. Model misspecification can be attributed to poor measurement of a variable or inclusion of inappropriate variables in the model. For example, Giordani (2004) showed that once the output is replaced by the output gap in the empirical model, the price puzzle vanishes at

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6 This view was also supported by Clarida, Gali and Gertler (1999).
least in the quarterly data. In order to examine the role of model indeterminacy problem, Kapinos (2011) extended the New Keynesian model by (i) distinguishing between the anticipated and unanticipated shocks to inflation, (ii) adding forward looking monetary policy, and (iii) including a cost channel of monetary policy transmission. Based on data generated from the model, Kapinos estimated an SVAR model. The empirical results based on this SVAR model suggest that overlooking the effect of anticipated shocks to inflation and forward-looking monetary policy behavior of the central bank in a standard Cholesky structural vector autoregressive identification scheme gives rise to the price puzzle. Furthermore, the cost channel of monetary policy transmission leads to price puzzle only in a misspecified model, where the anticipated shocks and forward looking monetary policy of the central bank are not accounted for.

2.3 Cost Channel of Monetary Policy and the Price Puzzle

Finally, another strand of the existing literature suggests that price puzzle arises due to the presence of a cost channel of monetary policy transmission. This channel is realized when an increase in nominal interest rate increases the cost of production. Within the context of developing countries, due to non-existence of well-functioning stock markets, cash starved domestic firms are effectively forced to finance their spending through bank loans at the market rate of interest, which results in a considerable cost channel. Barth and Ramey (2001) detected a strong cost channel in the pre-1979 industry level data from the US. However, Rabanal (2003) failed to detect a strong cost channel in the post 1984 US data. Rabanal’s simulation results revealed that, in order for the price puzzle to occur in the US economy, in the post 1984 period,

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7 However, Kapinos (2006) failed to confirm this result.
8 Ravenna and Walsh (2002), Blake and Kirsanova (2004) and Liosa and Tuesta (2009) are just a few examples of the studies where the supply side effect of the interest rate is explicitly taken into account.
the firms would have to borrow at a rate which was at least four times higher than the Federal Reserve rate.

Gaiotti and Secchi (2006) and Barth and Ramey (2001) argued that manufacturing sector firms respond to contractionary monetary policy shocks with high prices. Using quarterly data, Ravenna and Walsh (2006) confirmed the presence of a cost channel in the US economy. Tillman (2008) supported the presence of a strong cost channel in the US economy by showing that the difference between the actual and estimated inflation rate drops significantly when the Phillips curve was estimated after taking the cost channel into account.

In the next section, we develop a new Keynesians model, where the role of exchange rate pass through is highlighted.

2. The Model

We consider an open economy, where households consume a domestic and an imported (i.e., a foreign) good. The domestic good is produced by both domestic and foreign firms (located in the domestic economy). The domestic firms borrow money at the market rate to finance their cost of production, which creates a cost channel of monetary policy. While operating in the home country, foreign firms measure all of their cash flows in the units of the foreign currency, which creates the possibility of the exchange rate pass through. The equilibrium of the economy under consideration is described by two structural equations (a Phillips curve and an IS curve). The Phillips curve (see the Appendix for detailed derivation) relationship for the economy is as follows:

\[ \pi_i^h = \beta E_t^h \pi_{i+1} + \alpha_1 x_t + \alpha_2 q_t + \alpha_3 i_t + u_t, \]

where \( x_t, i_t, \pi_i^h \) and \( q_t \), respectively, are the output gap (i.e., the difference between the actual and natural output) at time \( t \), the short-term nominal interest rate, domestic inflation (
\[ \pi_i^h = p_i^h - p_{i-1}^h, \text{ where } p_i^h \text{ is the price of the domestic good}, \] and the real exchange rate (\[ q_i = e_i + p_{i}^f - p_i^h, \text{ where } e_i \text{ is nominal exchange rate and } p_i^f \text{ is the price of the imported good}; \]
\[ E_i^{*} \] is the expectations operator for private agents conditional on the time \( t \) information set; \( u_{it} \sim N(0, \sigma_{u_i}^2) \) is a white noise cost-push shock that is normally distributed with variance \( \sigma_{u_i}^2 \); \( \beta \) is the discount factor; \( \alpha_1, \alpha_2, \text{ and } \alpha_3 \) are the deep parameters as explained in the Appendix.

Since it is always costly for the firms to change goods prices, the representative firm chooses to gradually close the gap between the actual and desired level of prices over the horizon.\(^9\) The Phillips curve equation used in this paper includes both the cost channel of monetary policy and exchange rate pass through. In other words, both the interest rate and real exchange rate have a direct bearing on domestic and CPI inflation rates through the supply side of the economy. It is worth mentioning that exchange rate pass through tends to be relatively more significant during inflationary periods (see, for example, Takhtamanova, 2010 for details).

The IS curve of the economy is given by equation (2) as follows:
\[
\begin{align*}
x_i = E_i^* x_{i+1} - \delta_1 \left( i_i - E_i^* \pi_{i+1}^{CPI} \right) - \delta_2 \left( E_i^* q_{t+1} - q_i \right) + \delta_3 \left( x_i^f - E_i^* x_{i+1}^f \right) + u_{2i},
\end{align*}
\]
where \( \pi_{t+1}^{CPI} \) is the CPI inflation rate; \( i_i - E_i^* \pi_{i+1}^{CPI} \) is the real interest rate; \( E_i^* q_{t+1} - q_i \) is the depreciation rate of the real domestic currency exchange rate; \( E_i^* x_{i+1} \) is the expected output gap; \( x_i^f - E_i^* x_{i+1}^f \) is the gap between the current output of foreign firms and its expected value in period \( t+1 \); \( u_{2i} \sim N(0, \sigma_{u_2}^2) \) is a white noise demand-pull shock that is normally distributed with variance \( \sigma_{u_2}^2 \); and \( \delta_1, \delta_2, \text{ and } \delta_3 \) are the deep parameters.\(^{10}\)

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\(^9\) Calvo (1983) derived a similar aggregate supply curve by assuming monopolistically competitive market structure, where firms adjust prices sluggishly over time.

\(^{10}\) \( \delta_i = (1 - \varepsilon_i) \sigma \); \( \varepsilon_i = \varepsilon_i \left[ (1 - \varepsilon_i) \eta + \eta^f \varepsilon_i^f \right] \); \( \varepsilon_i = \varepsilon_i \beta^f \) where \( \sigma \) is the intertemporal elasticity of substitution, which is assumed to be positive; \( \varepsilon_i^f \) is the weight of the price of the imported good in CPI; \( \eta \) is the elasticity of substitution between the domestic and imported good; \( \eta^f \) is the elasticity of substitution between the domestic and imported good in the foreign country; and \( \beta^f \) is the foreign marginal propensity to consume.
Equation (2) is derived by maximizing the utility of the representative household subject to a standard budget constraint (see Guender, 2003 for a detailed derivation).

The log-linear version of the uncovered interest rate parity is defined as follows:

\[ i_t = i_{t}^f + E_t q_{t+1} - q_t + E_t \pi_{t+1}^h + u_{3t} \]  \hspace{1cm} (3)

where \( i_t^f \) measures the foreign rate of interest; \( u_{3t} \approx N(0, \sigma_{u3}^2) \) is a white noise risk premium shock that is normally distributed with variance \( \sigma_{u3}^2 \).

The CPI and domestic price \( p_t^h \) are defined as follows:

\[ p_t^{CPI} = (1 - e_b) p_t^h + e_b \left( e_t + p_t^f \right) \]  \hspace{1cm} (4)

\[ p_t^h = (1 - \mu) p_t^D + \mu p_t^F \]  \hspace{1cm} (5)

where \( e_b \) is the weight of the price of the imported (i.e., foreign) good in CPI; \( \mu \) is the proportion of foreign firms operating domestically (see the Appendix for details). The domestic price is the weighted average of the price charged by the domestic \( (p_t^D) \) and foreign firms in the domestic market \( (p_t^F) \).

Equations (6) to (8), respectively, explain how individuals form expectations about the output gap, domestic inflation\(^{11}\), and the nominal exchange rate.

\[ E_t x_{t+1} = a_t = a_{t-1} + \gamma_t (x_{t-1} - a_{t-1}) \]  \hspace{1cm} (6)

\[ E_t \pi_{t+1}^h = b_t = b_{t-1} + \gamma_t (\pi_{t-1}^h - b_{t-1}) \]  \hspace{1cm} (7)

\[ E_t e_{t+1} = c_t = c_{t-1} + \gamma_t (e_{t-1} - c_{t-1}) \]  \hspace{1cm} (8)

Given the widespread criticism of the idea of rational expectations (RE),\(^{12}\) we assume that agents form expectations based on adaptive learning (both constant and decreasing learning).

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\(^{11}\) The CPI and domestic inflation rates are linked as follows \( \pi_t^{CPI} = (1 - e_b) \pi_t^h + e_b (e_t - e_{t-1}) \). Throughout the paper, we assume that \( \ln p_t^f = p_t^f = 0 \).
According to the RE hypothesis, economic agents do not make systematic errors since they are aware of the probability distribution (or their subjective probability distribution coincides with the true distribution of the economy) of all the random shocks that hit the economy from time to time. The idea of RE is based on the assumption that each agent knows the structure of the model that describes the economy. Accordingly, by coordinating their expectations, economic agents can affect the aggregate outcomes (see Evans and Honkapohja, 2001 for details). The RE hypothesis is based on a number of strong assumptions that do not hold in real life. Accordingly, it is desirable to consider the alternatives to an RE based equilibrium. Using survey data to examine how expectations are formed, a number of studies (for example Evans and Gulamani, 1984; Bonham and Dacy, 1991; Croushore, 1997 and Bonham and Cohen, 2001) found the RE hypothesis to be unsatisfactory in the case of inflation expectations. Froot (1989), Friedman (1980) and Jeong and Maddala (1996) confirm the failure of the RE hypothesis in the case of interest rate forecasting. Within the context of exchange rate expectations, a number of experimental and survey data based studies have also rejected the RE hypothesis and instead supported the idea of adaptive, regressive, and distributed lag expectations (see Hodric, 1987; Froot and Thaler, 1990; and Marey, 2004).

The lack of empirical support for the RE hypothesis prompted economists and financial analysts to propose alternative mechanisms. For example, Mickiewicz and Pick (2013), while using real time data, report that constant gain learning models perform better for inflation, 12

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12 For example, see Marcet and Nicolini (2003), Millani (2007), and Slobodyan and Wouters (2012) for detail discussion.

13 Adaptive learning can take the form of constant gain or decreasing gain learning. Constant gain learning can be imposed in our model by assuming that $\gamma_t = \gamma \in (0,1) \forall t$, whereas a decreasing gain learning can be incorporated in the model, if we set, for example, $\gamma_t = t^{-1}$. A constant gain learning means that more recent observations are given a higher weight, whereas decreasing gain learning means that more recent observations are given the same weight (Santoro, 2006).
unemployment, and the yen-dollar exchange rates. In contrast, the decreasing gain learning models perform better in the case of interest rate, GDP growth, and the pound-dollar exchange rate. Similarly, Adam, Marcet and Nicolini (2016) and Evans and Branch (2010), among others, showed that adaptive learning hypothesis was more suitable in the case of financial markets. Furthermore, Preston and Eusepi (2013) and Milani (2011) showed that adaptive learning behavior was relatively more useful in explaining business cycle fluctuations.

The idea of adaptive learning is based on the assumption that agents, due to limited knowledge about the structure of the economy, do not know the exact time path that endogenous variables follow. However, we assume that agents are able to estimate the perceived law of motion (PLM) of these variables recursively as set by the central bank (see Orphanides and Williams, 2004; Evan and Honkapohja, 2010; and Molnar and Santoro, 2014 for details). This form of expectations implies bounded rationality. As far as the Lucas critique is concerned, Molnar and Santoro argue that, although the central bank can manipulate agents’ expectations, when expectation are based on adaptive learning, agents revise their expectations over time and make them consistent with central bank policies. In other words, in the case of adaptive learning, agents do not make systematic mistakes. In addition, Orphandies and Williams (2004) argue that models based on learning are free from the Lucas critique since agents’ expectations endogenously adjust to changes in policy. Finally, it is worth mentioning that these studies argue that model consistency and absence of mistakes are properties of learning models that hold asymptotically.

We assume throughout that the central bank has full information on the structure of the economy including the precise form through which agents form their expectations. We also
assume that the disturbance terms $u_{1t}$, $u_{2t}$, and $u_{3t}$ that capture the random shocks are independent of each other and satisfy the usual assumptions.

Before moving forward, it is imperative that we explain how the currency exchange rate and interest rates affect the economy under consideration. Our model captures the exchange rate effect through more than one channel. On the demand side, depending on the degree of openness of the economy, the exchange rate affects the net exports through changes in the relative competitiveness. Similarly, the exchange rate changes also affect the interest rate parity, which in turn affects the aggregate demand for goods via changes in the real interest rate. Theoretically, the exchange rate can exert negative supply-side effect in more than one ways. For example, if we assume that firms use imported inputs (such as oil) then exchange rate fluctuations would affect the cost of production. Similarly, given that foreign firms operate in the domestic economy, the exchange rate pass through can also affect goods prices. In any case, the exchange rate depreciation would have both positive (demand side) and negative (supply-side) effects. Hence, the net effect of any change in the domestic currency on the key macroeconomic variables (such as the output gap and inflation rate) depends on these competing effects.

Our model also captures both demand and supply-side effects of the interest rate. As the rate of interest increases, the aggregate demand decreases via decrease in consumption and investment. An increase in the interest rate also decreases the aggregate supply. In such a case, would a negative monetary shock reduce the rate of inflation? The answer to this question depends on the competing interest rate related demand and supply side effects.

This completes the description of the model. We now turn our attention to the question of how the central bank conducts the monetary policy under three monetary policy alternatives.
Given that the closed form analytical results cannot be derived, the usefulness of these policies will be evaluated by means of model calibrations.

3. Central Bank Monetary Policy Choices

Svensson (2003), among others, maintains that in practice the central banks do not follow some simple rules to conduct their monetary policy. In order to examine the sensitivity of our simulation results to central bank choices, we consider three monetary policy alternatives (the optimal monetary policy rule, money growth targeting rule, and a Calvo-type interest rate rule).

3.1 The Optimal Monetary Policy (OMP) Rule

Following the existing literature, we assume that monetary authority acts to maximize the following social welfare function (SWF) with respect to the CPI inflation ($\pi_t^{\text{CPI}}$), output gap, exchange rate and the interest rate subject to the constraints given by equations (1) to (8):

$$\text{Max}_{z} [\text{SWF}]=-\frac{1}{2}E_t\sum_{j=0}^{\infty} \beta^j \left[ \pi_{t+j}^{\text{CPI}} + w_x x_{t+j}^2 + w_i i_{t+j}^2 + w_e \left( e_{t+j} - e_{t+j-1} \right)^2 \right]$$

where

$$\pi_t^{\text{CPI}} = (1-\epsilon_b)\pi_t^h + \epsilon_b (e_t - e_{t-1})$$

$$z = p_t^h x_t, i_t, e_t, a_t, b_{t+1}, c_{t+1}$$

The above optimization problem yields the usual first order conditions (see the Appendix) that help us to trace the optimum time path of $\pi_t^{\text{CPI}}$, $\pi_t$, $i_t$, $e_t$, $a_t$, $b_t$, and $c_t$. The last expression in the parenthesis captures the interest rate smoothing policy of the central bank (also known as the past-dependent monetary policy). It is important to note that, along with other
variables, the central bank chooses $a_{t+1}$, $b_{t+1}$ and $c_{t+1}$ which suggests that the central bank is attempting to tame agent expectations.\textsuperscript{14} It will be interesting to see whether the price puzzle continues to hold when the central bank explicitly controls agents' inflation expectations from the outset. In the case of adaptive learning, the central bank anchors inflationary expectations through its credible track record of stable inflation. In real life, it is unlikely that central banks know how individuals form their expectations. However, it is interesting to see how central banks conduct their monetary policy if they were blessed with full information.

3.2 Money-Growth Targeting (MGT) Rule

In practice, the central banks of small open economies lack experience and technical knowhow needed to conduct a sophisticated monetary policy (such as the optimal monetary policy). Accordingly, irrespective of the cyclical behavior of the economy, economists like Milton Friedman strongly advocated the use of a simple constant growth of money supply rule.\textsuperscript{15} Ordinarily, the short-term interest rate is used to control money growth, which in turn helps to control inflation. Let $\Omega_t = m_t - m_{t-1}$ denote the money growth, where $m_t = \ln M_t$ and $M_t$ is money supply. Suppose that monetary authorities wish to control inflation by controlling the growth of money supply at a target rate of say $\hat{\Omega}$. The period loss ($L_t$) function of monetary authorities is given as follows:

\textsuperscript{14} As indicated earlier, following the existing literature, for example Gaspar et al. (2006), we assume that the central bank has full information about the structure of the economy including how agents form their expectations.

\textsuperscript{15} Money growth targeting rule is regarded as an alternative to inflation targeting rule. Svensson (2010, p. 5) argues that money-growth targeting has been tried by many countries in the past but they did not continue with the policy because of the lack of a consistent relationship between money growth and inflation. Furthermore, Svensson noted that many small and medium-sized countries have tried exchange rate targeting as a means to stabilize both inflation and the economy but could not continue with this policy probably due to the difficulties involved in defending the misaligned fixed exchange rate.
\[ L_t = 0.5 \left( \Omega_t - \hat{\Omega} \right)^2 \]  

Differentiating equation (10) with respect to \( \Omega_t \) and applying the expectations operator, we get

\[ E_t \Omega_{t+T} = \hat{\Omega} \]  

where \( T \geq 0 \) is the shortest control lag of the growth of money supply.

Svensson (2011) termed this strategy as "money-growth forecast targeting" and suggested the use of a monetary policy instrument (such as the short-term interest rate) to achieve this goal. The relevant policy instrument can be derived by assuming a money demand function as follows:

\[ m_{t+1} - p_{t+1}^{CPI} = \phi_s x_t - \phi_i (i_t - \bar{T}) - \xi_{t+1} \]  

where \( \phi_s, \phi_i > 0 \) and \( \bar{T} \) is the steady state level of the short-term interest rate.

We further assume that \( \xi_{t+1} \) is an iid variable with zero mean and a constant variance. Using equations (11) and (12) and after some manipulations, we can derive the following expression for money supply growth rate target:

\[ \hat{\Omega} = E_t \pi_{t+1}^{CPI} + \phi_s (x_t - x_{t-1}) - \phi_i (i_t - i_{t-1}) + \xi_t \]  

Finally using equation (13) and the standard Taylor rule, the augmented Taylor rule can be specified as follows:

\[ i_t = \rho i_{t-1} + \left( 1 - \rho \right) \left[ \zeta_x \pi_t^{CPI} + \zeta_x x_t + \zeta_{\Omega} \left( E_t \pi_{t+1}^{CPI} + \phi_x (x_t - x_{t-1}) - \phi_i (i_t - i_{t-1}) \right) \right] + u_{t} \]  

where \( u_t = \zeta_{\Omega} \xi_t \)

The above interest rate rule has some very interesting properties. For example, the central bank is forward-looking and sets the interest rate based on expected future CPI inflation rate.
Furthermore, while setting the interest rate, the central bank also cares about the output growth rate (also known as the speed limit targeting due to Walsh, 2003) i.e., $x_t - x_{t-1}$.

### 3.3 A Calvo-Type Taylor Rule (CTR)

Following Kapinos (2011), which is based on Levine et al. (2007), we assume that the central bank sets the nominal interest rate as a weighted average of the target nominal interest rate ($i^*_t$) and the lagged interest rate as follows:

$$i_t = (1 - \rho) E_{t-1} i^*_t + \rho i_{t-1} + u_{st} \tag{15}$$

Following Levine et al., we further assume that the target nominal interest rate is set using the following relationships

$$i^*_t = \nu_i \Theta_t + \nu_x x_t \tag{16}$$

$$\Theta_t = (1 - \phi) E_t \sum_{j=0}^{\infty} (\phi^j \pi_{t+j}); \ 0 < \phi < 1 \tag{17}$$

where $\Theta_t$ is the discounted sum of the future expected inflation rates and $\phi$ measures the degree of central bank forward looking behavior.

Gabriel et al. (2009) combined equations (15) to (17) to derive equation (18) as follows, which is also known as a Calvo-type Taylor interest rate rule.

$$i_t = \left(\frac{\rho}{1 + \rho \phi}\right) i_{t-1} + \left(\frac{1 - \rho}{1 + \rho \phi}\right) E_{t-1} \left(\frac{\phi}{1 - \rho} i_{t+1} + \nu_x (1 - \phi) \pi_t + \nu_i (x_t - \phi x_{t+1})\right) + u_{st} \tag{18}$$

The above rule resembles the traditional Taylor rule, where expectations play no role in determining the interest rate, if we set $\phi = 0$ in equation (18). However, if we set $0 < \phi < 1$, the above rule becomes a Taylor rule, which is also known as the speed limit target (SLT), proposed by Walsh (2003). Orphandies (2003) suggests that the presence of output gap growth rate
\((x_{t+1} - x_t)\) improves the estimation of the Taylor rule with real data. The presence of \(i_{t-1}\) term implies that the central bank follows the policy of interest rate smoothing.

4. Model parameterization and calibration

4.1 Model parameters

We start our calibration exercise by assuming the following parameter values:\(^{16}\)

Table 1: Parameter description and assumed values

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\beta')</td>
<td>Foreign marginal propensity to consume</td>
<td>0.9</td>
</tr>
<tr>
<td>(d)</td>
<td>Ratio of the costs of changing prices to the costs of being away from the optimal price</td>
<td>5</td>
</tr>
<tr>
<td>(\sigma)</td>
<td>Coefficient of relative risk aversion</td>
<td>1</td>
</tr>
<tr>
<td>(\phi_s)</td>
<td>Income elasticity of money demand</td>
<td>1</td>
</tr>
<tr>
<td>(\phi_i)</td>
<td>Interest elasticity of money demand</td>
<td>0.01</td>
</tr>
<tr>
<td>(v)</td>
<td>Cost function parameter, which measures the proportionate change in total cost when output increase by 1%.</td>
<td>1.05</td>
</tr>
<tr>
<td>(\phi)</td>
<td>Degree of Central Bank forward-looking behavior</td>
<td>0.75</td>
</tr>
<tr>
<td>(\theta)</td>
<td>Interest rate pass through</td>
<td>0-1.75</td>
</tr>
<tr>
<td>(\beta)</td>
<td>Rate of discount</td>
<td>0.9</td>
</tr>
<tr>
<td>(\rho)</td>
<td>Interest-smoothing-parameter</td>
<td>0.70</td>
</tr>
<tr>
<td>(\zeta)</td>
<td>Magnitude of the response to inflation target in the Taylor rule</td>
<td>1.5</td>
</tr>
<tr>
<td>(\zeta_s)</td>
<td>Magnitude of the response to output gap in the Taylor rule</td>
<td>0.5</td>
</tr>
<tr>
<td>(\zeta_{\Omega})</td>
<td>Magnitude of the response to money supply growth rate target in the Taylor rule</td>
<td>0.25</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>Constant gain learning</td>
<td>0.5</td>
</tr>
<tr>
<td>(\eta)</td>
<td>Elasticity of substitution between the domestic and imported good</td>
<td>1</td>
</tr>
<tr>
<td>(\eta')</td>
<td>Elasticity of substitution between the domestic and imported good in the foreign country</td>
<td>2</td>
</tr>
<tr>
<td>(\varepsilon_s)</td>
<td>Weight of the price of the imported good in the CPI</td>
<td>0.25</td>
</tr>
<tr>
<td>(w_1)</td>
<td>Weight of the output gap in SWF loss function</td>
<td>0.5</td>
</tr>
<tr>
<td>(w_2)</td>
<td>Weight of the interest rate in SWF loss function</td>
<td>0.1</td>
</tr>
<tr>
<td>(w_3)</td>
<td>Weight of the interest rate smoothing in SWF loss function</td>
<td>0.01</td>
</tr>
<tr>
<td>(\mu)</td>
<td>Degree of exchange rate pass-through</td>
<td>0.025, 0.625</td>
</tr>
<tr>
<td>(\varepsilon)</td>
<td>Absolute value of the elasticity of demand for domestic good</td>
<td>1.34</td>
</tr>
<tr>
<td>(\nu_s)</td>
<td>Magnitude of the response to output target in a CTR model</td>
<td>1</td>
</tr>
<tr>
<td>(\nu_s)</td>
<td>Magnitude of the response to inflation target in a CTR model</td>
<td>2</td>
</tr>
</tbody>
</table>

The parameter values presented in Table 1 are pretty standard. Theoretically, the value of \(d\) (which measures the ratio of the costs of changing prices to the costs of being away from the

\(^{16}\) The calibration results presented in this paper are based on the assumption of constant gain learning. Our results also hold in the case of decreasing gain learning.
optimal price lies in the rage of zero to infinity (see Foryen and Guender, 2007, p. 270 for details). If $d = 0$ then firm always charges the ideal price. When $d$ approaches infinity then the Phillips curve equation reduces to $E_t \pi_{t+1} = \beta^{-1} \pi_t$ (see the Appendix for details). Guender (2003) implicitly sets $d = 4$ to get realistic coefficients in the Phillips curve equation. Following the same approach and considering the stability of the model under different policy alternatives, we set $d = 5$. Values of $\delta_1$, $\delta_2$ and $\delta_3$ are determined endogenously by other parameters, such as $\varepsilon_b$, $\sigma$, $\pi_f$, $\beta^f$, and $\varepsilon_f^f$ that are borrowed from Guender (2003). Similarly, the values of $\alpha_1$, $\alpha_2$, and $\alpha_3$ are determined endogenously (see equations A10 to A13 in the Appendix).

The parameter $\alpha_3 = [\theta(1-\mu)k_3]d^{-1}$, where $k_3 = [1+\varepsilon(1-1)]^{-1}$, measures the extent of the cost channel. If $\theta = 0$ then the interest rate has no bearing on inflation by way of supply side. In Rabanal (2003), the price puzzle arises only when $\theta$ takes a value of 4. Whereas, Ravenna and Walsh (2003) estimated $\theta$ to be as high as 11. Rabanal (2003, page 31) argues that $\theta = 11$ implies that if the Federal funds rate was around 6 percent then domestic firms must be borrowing at an effective interest rate of 66 percent. In our simulation exercise, we use a very moderate value of $\theta$ (i.e., 1.75).

The other parameter which plays an important role in resolving the price puzzle is $\alpha_2$. As shown in the Appendix, $\alpha_2 = \mu k_3 d^{-1}$, where $0 < \mu \leq 1$, measures the degree of exchange rate pass through. If $\mu = 0$ then there is no exchange rate pass through since all firms are domestic firms and there is no incentive for them to adjust the prices in response to exchange rate fluctuations. A higher value of $\mu$ represents a higher degree of exchange rate pass through. Finally, it is important to note that the cost channel also depends on the degree of exchange rate fluctuations.
pass through, i.e., $\alpha_3 = \theta(1 - \mu)[1 + \varepsilon(\nu - 1)]^{-1}$. When the exchange rate pass through is complete (i.e., $\mu = 1$), the cost channel is completely ineffective. From the expression for $\alpha_3$, we note that as the degree of exchange rate pass through increases, the impact of the cost channel of monetary policy diminishes and vice versa. The choice of $\lambda_x, w_1$, and $w_2$ as shown in Table 1 is based on Svensson (2000). We set $\phi = 0.01$, and $\phi_\pi = 1$, which is consistent with related studies. The rest of the parameter values are borrowed from Kapinos (2011). Finally, in the rest of this paper, we assume that the log of foreign real output $x_t^f = 0$.

4.2 The Impact of a Monetary Shock

In order to examine the relationship between the interest rate and inflation, we assume that the disturbance terms in the model are white noise. We further assume that that monetary shock always comes as a surprise. However, before we report our simulation results, it is useful to briefly outline a few results that one expects in the aftermath of a contractionary monetary policy shock. We expect that a contractionary monetary policy shock will increase the nominal interest rate in both the short and the medium runs. As money supply decreases, we expect inflation to fall in both the short and the medium runs. As prices adjust sluggishly, we expect the output to fall. We expect both consumption and investment demand to fall and the exchange rate to appreciate both in the short and the medium runs. However, it is also possible that the exchange rate immediately over shoots and then reverts to its long run value over time. Finally, because prices adjust sluggishly over time, we expect that appreciation of the nominal exchange rate leads to appreciation of the real exchange rate. Thus within the context of this paper, an increase in the interest rate and
appreciation of the exchange rate (both nominal and real) play an important role in explaining and resolving the price puzzle.

We start our calibration exercise by assuming that a cost channel to monetary policy transmission is present but there is no exchange rate pass through, which implies that there are no foreign firms in the economy. We then consider the case where the cost channel is present and some degree of exchange rate pass through exists. The simulation results are presented in Figure 1, where the impact of a 1 standard deviation interest rate shock on four key economic variables (i.e., $i_t, \pi_t^{CPI}, \pi_t^{H}, x_t$, and $q_t$) is shown. Each panel in Figure 1 shows three time paths (each associated with one of the three monetary policy alternatives that the central bank can use).

![Image of Figure 1](image-url)

**Figure 1**: Impulse response to monetary shock under three monetary policy rules ($\theta = 1.75, \mu = 0$)

**Notes**: CTR is a Calvo-type Taylor Rule; MGT is money growth targeting; OMP is the optimal monetary policy; Domestic inflation is the inflation rate based on a general price index.
Figure 1 shows that, in the absence of exchange rate pass through (i.e., when $\mu = 0$), a given monetary policy shock gives rise to the price puzzle under all three monetary policy alternatives. Figure 1 shows that a contractionary monetary shock increases the nominal interest rate, which leads to (i) a decrease in the output and (ii) exchange rate appreciation. As there is no exchange rate pass through (because $\mu = 0$), the impact of the cost channel dominates the contractionary demand side effect, which results in an increase in both domestic and CPI inflation rates (domestic inflation rate is calculated using a general price level). It is interesting to note that domestic inflation rate increases immediately and then peter out temporally. On the other hand, the CPI inflation rate follows a hump shape, i.e., increases gradually over a period of time, reaches a maximum and then converges back to its original value sluggishly. As far as the behavior of the real exchange rate is concerned, in the case of both CTR and MGT, the real exchange rate follows a hump shape pattern. However, in the case of the OMP, the trajectory of the real exchange rate is consistent with Dornbusch (1976) model (i.e., the currency appreciates immediately and then gradually depreciates over time).

Figure 1 also shows that that the output gap increases relatively more in case of a CTR and MGT but relatively less in the case of OMP. The comparative loss in the output is lower and moderate changes in inflation occur in case of the OMP, which is not surprising. Since we assume that agents form expectations based on adaptive learning, the central bank’s job of anchoring the expectations of household becomes much easier.

Figure 1 also shows that a contractionary monetary shock leads to an increase in the real interest rate and currency appreciation. Our results are consistent with Bjørnland (2008 & 2009). Bjørnland’s simulation results show that both the exchange rate and real interest rate effects are
sufficiently strong to decrease the aggregate demand for goods, which exerts a negative pressure on both the output gap and inflation. In the absence of a cost channel to monetary policy (i.e., $\theta=0$), a monetary policy shock leads to a decrease in both the output and inflation and there is no price puzzle. In order to conserve space, these results are not reported here. However, these results are available to the readers upon request.

We now turn to the situation where exchange rate has some degree of pass through (e.g., $\mu = 0.25$) and continue to assume that a cost channel to monetary policy transmission is present. The model simulation results are shown in Figures 2a and 2b. Figure 2a shows that, in the presence of exchange rate pass through, a contractionary monetary shock increases the interest rate but the CPI inflation falls under all three monetary policies alternatives. However, the price puzzle continues to hold when a general price index is used as a measure of inflation under all the three monetary policy alternatives. When the weight of the price of the imported good is low (e.g., $e_b = 0.05$), the price puzzle emerges even when the CPI is used as a measure of inflation. However, when the exchange rate pass through is high (e.g., $\mu = 0.65$), the price puzzle disappears even when the general price index is used as a measure of inflation. To conserve space, these results (i.e., when the exchange rate pass through is high) are not reported in this paper.
Figure 2a: Impulse response to monetary shock under three monetary policy rules \((\theta = 1.75, \mu = 0.25)\).

Notes: CTR is a Calvo-type Taylor Rule; MGT is money growth targeting; OMP is the optimal monetary policy; Domestic inflation is the inflation rate based on a general price index.
OMP, MGT, and CTR variables return to their steady state values in the run; Domestic inflation is the inflation rate based on a general price index.

Figure 2b: Impulse response to monetary shock under three monetary policy rules ($\theta = 1.75, \mu = 0.65$).

Notes: CTR is a Calvo-type Taylor Rule; MGT is money growth targeting; OMP is the optimal monetary policy; Domestic inflation is the inflation rate based on a general price index.

The intuition behind the results presented in Figures 2a and 2b is as follows. A contractionary monetary shock increases the interest rate and appreciates the real exchange rate. However, the increase in the interest rate along with currency appreciation depresses the aggregate demand for goods, which puts a negative pressure on both the output gap and inflation. In the presence of a sufficiently high exchange rate pass through, currency appreciation can offset the effect of the cost channel. As a result, in Figure 2a, we observe a decrease in the CPI inflation as well as a decrease in output (i.e., an increase in the negative output gap) both in the short and the medium-run before these variables return to their steady state values in the long run. However, as we allow a higher degree of exchange rate pass through, even in the case of general price index based inflation, the price puzzle disappears as shown in Figure 2b. Our results are partly consistent with Bjørnland (2008), where price puzzle disappears in the case of the Norwegian economy when exchange rate is explicitly taken into account.17

It is perhaps worth mentioning that calibration results presented in this paper also suggest that a decrease in inflation is associated with a cost to the society in the form of a decrease in the

17 Bjørnland (2008) shows that, in the event of a contractionary monetary policy shock, the real exchange rate instantly appreciates before gradually reverting to its steady state value over time.
aggregate output. However, the output cost of a decrease in inflation is the lowest when the central bank uses the optimal monetary policy rule.

4.3 Sensitivity analysis

In this section we check the robustness of our results with respect to rational expectations hypothesis.

Figures 3a and 3b below, show that our main result is not sensitive to how agent expectations are formed. The results of model calibration under rational expectations are qualitatively similar to those derived under adaptive learning. It is worth mentioning that, under RE, a higher level of variability is evident in the general price index based domestic inflation whereas, under adaptive learning, we observe a higher level of volatility in CPI based inflation. The choice of adaptive learning in this paper is based on the real life applicability of the model. Using a theoretical model, where agents form expectations rationally and perfect international risk sharing exists, Ali and Anwar (2016) showed that price puzzle occurs in an open economy, with no exchange rate pass through, only when a cost channel to monetary policy is present. They also showed that the likelihood of price puzzle varies inversely with the degree of exchange rate pass through. In this paper, assuming adaptive learning, we present the results of model calibration that support the theoretical predictions of Ali and Anwar.
5. Concluding remarks

Using a medium scale New Keynesian dynamic stochastic general equilibrium (DSGE) model, this paper focuses on the role of exchange rate pass through in solving the price puzzle. Unlike the existing literature, we assume that agent expectations are based on adaptive learning and three monetary policy alternatives are available to the central bank (the optimal monetary
policy, money growth targeting rule, and a Calvo-type interest rate rule). In addition, we use two alternative measures of inflation (the consumer price index based inflation rate and a general price index based inflation rate, where the latter is referred to as the domestic inflation rate). Since our model includes some forward looking variables, it is not possible to derive closed form analytical results and hence, using plausible parameter values, we rely on model calibration. In addition to deriving some new results, this paper also provides empirical support for some existing theoretical predictions.

Assuming that there is no exchange rate pass through but a cost channel of monetary policy transmission exists, we show that the presence of a cost channel is a necessary condition for price puzzle to occur under all three monetary policy rules. In the presence of a moderate degree of exchange rate pass through, in response to a contractionary monetary policy shock, the CPI inflation falls but domestic inflation continues to increase under all three monetary policies alternatives. However, irrespective of how inflation is measured, in the presence of a sufficiently strong exchange rate pass through, the price puzzle disappears. We also considered the impact of a contractionary monetary policy on the output gap and the real exchange rate under all three monetary policy rules. We find that the decrease in aggregate output is in the lowest when the central bank uses the optimal monetary policy.

Based on the results presented in this paper, it can be argued that, in the absence of a sufficiently strong exchange rate pass through, a contractionary monetary policy may not succeed in reducing the rate of inflation. Furthermore, a contractionary monetary policy shock may also make the output gap more volatile.
Acknowledgements

This paper has greatly benefitted from very helpful comments and suggestions from two anonymous reviewers. We are also thankful to Pavel Kapinos for helpful comments on an earlier draft of this paper. However, the authors are solely responsible for all remaining errors and omissions.

Appendix

A.1 Derivation of Equation 1

Derivation of equation (1) is based on Froyen and Guender (2007) and Takhtamanova (2010). We assume throughout that two types of firms (namely domestic firms and foreign firms) operate in the economy. We also assume that price adjustment is always costly for the firms. A typical domestic firm $i$, chooses price $p_i^D$ while a typical foreign firm $j$, chooses $p_j^F$ such that the cost of being away from the optimal price is minimum. For instance, if the optimal price for the domestic firm is $p^{D*}$ then firm always wants to minimize the following expected cost function by choosing $p_i^D$.

$$
\min Z_i = E\sum_{\tau=1}^{\infty} \beta^{\tau-1} \left[ \left( p^D_i - p^{D*} \right)^2 + d \left( p^D_i - p^D_{i-1} \right)^2 \right]
$$

\begin{equation}
(A1)
\end{equation}

where $Z_i$ measures total cost of firm $i$ at time $t$; $\beta$ is the discount factor; and $d$ measures the cost of adjustment of prices.

Differentiating the above function with respect to $p_i^D$, we get the following expression:

$$
\beta dE_i \pi_i^D = p_i^D - p_i^{D*} + d \pi_i^D
$$

\begin{equation}
(A2)
\end{equation}

Similarly, for the foreign firm we can derive the following expression.

$$
\beta dE_i \pi_i^F = p_i^F - p_i^{F*} + d \pi_i^F
$$

\begin{equation}
(A3)
\end{equation}
The desirable price charged by the domestic firm is obtained by solving the following profit maximizing problem.

$$\Pi_d = P_D^d X_d - C_d$$

(A4)

The real profit of a representative domestic firm is as follows:

$$\frac{\Pi_d}{P_t} = \frac{P_D^d X_d}{P_t} - \frac{C_d}{P_t}$$

(A5)

We define our domestic price index ($P_d^d$) as follows:

$$P_d^h = \mu P_D^d + (1 - \mu) P_t^F$$

(A6)

To proceed further, we use demand and cost functions as follows:

$$X_d = X_{it} \left( \frac{P_D}{P_t^h} \right)^{-\varepsilon}; \varepsilon > 1$$

(A7)

$$C_d = \left( \frac{\varepsilon - 1}{\nu \varepsilon} \right) X_{it}^{\nu} (1 + i)^{\theta}; \nu > 1; \theta > 1$$

(A8)

where $\varepsilon$ is the absolute value of the elasticity of demand for the domestic good, which also enters the cost function of the representative firm for mathematical simplicity.

Substituting (A7) and (A8) into (A5), we get the following equation.

$$\frac{\Pi_d}{P_t^h} = \pi_d = X_{it} \left[ \left( \frac{P_D^d}{P_t^h} \right)^{-\varepsilon} - \left( \frac{\varepsilon - 1}{\nu \varepsilon} \right) \left( \frac{P_D^d}{P_t^h} \right)^{-\varepsilon} \right] (1 + i)^{\theta} X_{it}^{\nu}$$

(A9)

Differentiating the above function with respect to $P_D^d$ and setting the result equal to zero, we get the following ideal price for the domestic firm.

$$P_{it}^{*d} = P_t^h + k_1 X_{it} + k_2 i$$

(A10)

where $k_1 = \frac{\nu - 1}{1 + \varepsilon (\nu - 1)}$ and $k_2 = \frac{\theta}{1 + \varepsilon (\nu - 1)}$
Except for the rate of interest, all variables are expressed in log form. The cost function of the foreign firm operating in the domestic economy is as follows:

$$C_{jt}^F = \left(\frac{\epsilon - 1}{\nu \epsilon}\right)^{\theta} (1 + r^f)^{\theta} X_{jt}^{\nu} Q_t$$

(A11)

where $Q_t = \frac{E_t^f P^f}{P_t^h}$ is the real exchange rate.

The exchange rate is used to measure the cost of production in the units of local currency.

The demand function for the goods produced by foreign firms is $X_{jt}^F = X_{jt}^f \left(\frac{P^f}{P_t^h}\right)^{-\epsilon}$. The ideal price corresponding to this demand function is as follows:

$$p_{jt}^{*f} = p_t^h + k_1 x_t + k_3 \left(p^f + e_t - p_t^h\right) + k_4 i_t^f$$

(A12)

where $k_1 = \frac{\nu - 1}{1 + \epsilon (\nu - 1)}$; $k_3 = \frac{1}{1 + \epsilon (\nu - 1)}$; $k_4 = \frac{\theta}{1 + \epsilon (\nu - 1)}$; and $Q = p^f + e_t - p_t^h$

As indicated earlier, except for the rate of interest, all variables are expressed in log form. We consider a symmetric equilibrium, where $p_{jt}^{*f} = p_t^f$ and $p_{jt}^{*d} = p_t^d$. Assuming $i_t^f = 0$, multiplying equation (A2) by $1 - \mu$ and (A3) by $\mu$ and using (A6), (A10) and (A11) and after some manipulations, we get the Philips curve as specified in equation (1).

$$\pi_t^h = \beta E_t^{\beta} \pi_{t+1}^h + \alpha_1 x_t + \alpha_2 q_t + \alpha_3 i_t + u_t$$

(A13)

where $\alpha_1 = k_1 d^{-1}$, $\alpha_2 = \mu k_3 d^{-1}$, $\alpha_3 = \theta (1 - \mu) k_3 d^{-1}$

A.2 Optimal Monetary Policy-Variable Inflation Targeting

The central bank minimizes the social welfare loss (SWL) by the choosing the optimal time path of variables as follows:
\[
\begin{align*}
\text{Max} & \left[ \text{SWF} \right] = -E \sum_{j=0}^{\infty} \beta^j \left\{ \pi_{t+j}^{\text{CPI}} + w_x x_{t+j}^2 + w_s i_{t+j}^2 + w_s \left( i_{t+j} - i_{t+j-1} \right)^2 \right\} + \\
\lambda_{t+j}^{\pi} & \left[ \beta b_{t+j} + \alpha_1 x_{t+j} + \alpha_2 \left( e_{t+j} - p^h_{t+j} \right) + \alpha_3 i_{t+j} + u_{t+j} - \pi^h_{t+j} \right] + \\
\lambda_{t+j}^c & \left[ a_{t+j} - \delta_i \left( i_{t+j} - \left(1 - \epsilon_B \right) b_{t+j} - \epsilon_B \left( c_{t+j} - e_{t+j} \right) - r^f_{t+j} \right) + \delta_2 \left( b_{t+j} - c_{t+j} + e_{t+j} \right) + u_{t+j} - x_{t+j} \right] + \\
\lambda_{t+j}^i & \left[ i_{t+j} - e_{t+j} + u_{t+j} - i_{t+j} \right] + \lambda_{t+j}^a \left[ a_{t+j} + \gamma_{t+j} \left( x_{t+j} - a_{t+j} \right) - a_{t+j} \right] + \\
\lambda_{t+j}^b & \left[ b_{t+j} + \gamma_{t+j} \left( \pi_{t+j}^h - b_{t+j} \right) - b_{t+j} \right] + \\
\lambda_{t+j}^c & \left[ c_{t+j} + \gamma_{t+j} \left( e_{t+j} - c_{t+j} \right) - c_{t+j} \right]
\end{align*}
\]

where

\[
\pi_{t+j}^{\text{CPI}} = (1 - \psi) \left( p^h_t - p^h_{t+j} \right) + \psi \left( e_t - e_{t+j} \right)
\]

\[
z = p^h_t, x_t, i_t, e_t, a_{t+j}, b_{t+j}, c_{t+j}
\]

where \( \lambda_{t+j}^{\pi}, \lambda_{t+j}^a, \lambda_{t+j}^b, \lambda_{t+j}^c, \lambda_{t+j}^i, \lambda_{t+j}^i, \) and \( \lambda_{t+j}^i \) are the langrage multipliers.

After differentiating equation (A14) with respect to \( a_{t+j}, b_{t+j}, c_{t+j}, p^h_t, x_t, i_t, \beta, \alpha, \lambda_{t+j}^i, \lambda_{t+j}^b, \lambda_{t+j}^c, \lambda_{t+j}^i, \) and setting the results equal to zero, we get the following first-order conditions:

\[
\left(1 - \epsilon_B\right) \pi_{t+j}^{\text{CPI}} = \beta \left(1 - \epsilon_B\right) E \pi_{t+j}^{\text{CPI}} - \left(1 + \alpha_2\right) \lambda_{t+j}^c + \beta E_i \lambda_{t+j}^c + \gamma \lambda_{t+j}^b - \gamma \beta E_i \lambda_{t+j}^b
\]

(A15)

\[
w_s x_t = \alpha_1 \lambda_{t+j}^c - \lambda_{t+j}^c + \gamma \lambda_{t+j}^a
\]

(A16)

\[
i_t = \left[ w_s i_{t+j} + w_s E_i i_{t+j} - \delta \lambda_{t+j}^c - \lambda_{t+j}^c + \gamma \lambda_{t+j}^c \right][w_s + \gamma \left(1 + \beta\right)]
\]

(A17)

\[
\left(1 - \epsilon_B\right) \lambda_{t+j}^i = (\alpha_2 + \epsilon_B) \lambda_{t+j}^c - \epsilon_B \beta \lambda_{t+j}^c - \epsilon_B \left(1 - \epsilon_B\right) (\delta_1 - \delta_2) \lambda_{t+j}^c \\
+ (1 - \epsilon_B) (1 - \gamma) \lambda_{t+j}^c - \epsilon_B \beta \lambda_{t+j}^b + \epsilon_B \beta \gamma \lambda_{t+j}^b
\]

(A18)

\[
\lambda_{t+j}^a = \beta \lambda_{t+j}^c + (1 - \gamma) \beta \lambda_{t+j}^c
\]

(A19)

\[
\lambda_{t+j}^b = \beta \lambda_{t+j}^c + \beta \left[ \delta (1 - \epsilon_B) + \delta \epsilon_B \right] \lambda_{t+j}^c + (1 - \gamma) \beta \lambda_{t+j}^c
\]

(A20)

\[
\lambda_{t+j}^c = \beta \lambda_{t+j}^c + \epsilon_B (\delta_1 - \delta_2) \beta \lambda_{t+j}^c + (1 - \gamma) \beta \lambda_{t+j}^c
\]

(A21)
\[ \pi^h_t = \beta E_t b_{t+1} + \alpha_i x_t + \alpha_2 \left( e_t - p^h_t \right) + \alpha_j i_t + u_{it} \] (A22)

\[ x_t = E_t a_{t+1} - \delta i_t \left[ 1 - \varepsilon \right] E_t b_{t+1} - \varepsilon \left( E_t c_{t+1} - e_t \right) + \gamma \left( E_t r_{t+1} - E_t c_{t+1} + e_t \right) + u_{2t} \] (A23)

\[ i_t = i^f + E_t c_{t+1} - e_t + u_{3t} \] (A24)

References


**Highlights**

A small-scale New Keynesian model is used to examine the price puzzle phenomenon
Agent expectations are based on adaptive learning
Model calibration results show that exchange rate pass through can resolve the puzzle
The main result holds under three alternative monetary policy rules
A decrease in inflation leads to a cost to the society in the form of lower output