Pre-service Teachers' Pedagogical Content Knowledge: Implications for Teaching

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Effective teachers have good pedagogical content knowledge (PCK). Pedagogical content knowledge is the intersection of discipline specific content knowledge and pedagogical knowledge. How effectively are pre-service teachers helped to develop good PCK? In this project we asked our pre-service teachers how they would respond to a particular student misconception before and after teaching three topics, to determine if there had been any growth in their PCK. Although the pre-service teachers had deepened their knowledge on teaching specific mathematics content, few changed their answer to the question or showed a deeper understanding of what the student had understood. This then has implications for our teaching - we need to make our thinking explicit so that pre-service teachers can see the complexity of these issues.

It made very little difference! I showed them different models and they played with them, we looked at samples of students work and discussed misconceptions and ways to move the students forward in their thinking and there is almost no changes in the pre-service teachers' thinking. What have I done wrong?" This statement was made whilst the authors were analysing and discussing pre-service teachers' responses to the question in Figure 1.

Background

Schulman (1986) believed that teachers needed more than just knowledge of their content area and generalised knowledge of pedagogy to be a good teacher. He believed that they also needed "the ways of representing and formulating the subject that make it comprehensible to others" (Schulman, 1986, p. 9) as well as what makes the concept easy or difficult for others and possible misconceptions that students may have.

Ball (2000) proposed that when preparing pre-service teachers we need to identify the important content knowledge needed for teaching, how that knowledge needs to be understood, and how that knowledge is actually learnt in the classroom. The intersection of discipline specific content knowledge and pedagogical knowledge is known as pedagogical content knowledge (PCK) and includes knowing what representations to use to make the concept accessible to learners and how to adapt tasks to suit where the students are at in their understanding. "Pedagogical content knowledge highlights the interplay of mathematics and pedagogy in teaching" (Ball, 2000, p. 245) . Hence PCK includes knowledge of: mathematics both conceptual and procedural; pedagogy; students; and curriculum (Kilic, 2009)

When teachers hold class discussions (be they mathematics teachers in schools or teacher educators in universities) they need to decide which students' ideas to follow up and pursue and which to let slide. Whilst students are working on problems, "The teacher formulates probes, pushes students, offers hints, and provides explanations. Students get stuck: What does one do to help them remobilize?" (Ball, p. 243). Lampert (1990, p. 41) describes her role in the mathematics classroom as:

In V. Steinle, L. Ball & C. Bardini (Eds.), Mathematics education: Yesterday, today and tomorrow (Proceedings of the 36th annual conference of the Mathematics Education Research Group of Australasia). Melbourne, VIC: MERGA.

> The role I took in classroom discourse, therefore, was to follow and engage in mathematical arguments with students; this meant that I needed to know more than the answer or the rule for how to find it, and I needed to do something other than explain to them why the rules worked. I needed to know how to *prove* it to them, in the mathematical sense, and I needed to be able to evaluate their proofs of their own mathematical assertions. In the course of classroom discussions, I also initiated my students into the use of mathematical tools and conventions. Information about tools and conventions was integrated with teaching the class about the process of doing mathematics.

This then is one of the difficulties of learning to teach and one of the challenges for pre-service teacher educators. An effective mathematics teacher needs to be able to make explicit to her/his students the mathematics that she/he is using to evaluate arguments or solutions, as well as be able to follow the students' arguments as they participate in discussions and develop their own solutions (Lampert, 1990). To do this successfully takes more than just mathematical knowledge and generic teaching skills (Ball, 2000). "In teaching there are many opportunities and examples that have the potential to be applied in pedagogically useful ways, and yet are not because the teacher does not perceive the opportunity" (Chick, 2007 p. 5). The classroom is also a complex learning environment with many competing factors that involve "a complex interplay between what *could be* possible, what *is* possible, and what is *seen as* possible" (Watson, 2003, p . 37). Consequently, teaching mathematics, where you nurture students' learning to become mathematicians, is difficult, to do well particularly when the teacher is not a mathematician.

Chick (2007) believes teacher education and professional development "must be more explicit about issues associated with example use" (p. 18). This will help with what Ball (2000) describes as the inability of many teachers "to hear students flexibly, represent ideas in multiple ways, connect content to contexts effectively, and think about things in ways other than their own."(p. 243) That is, we need to spend time with pre-service teachers doing specific examples and investigating the pedagogical implications of what can be done with these examples. Kilic (2009) attempted to determine the development of pre-service teachers' PCK in a methods course and associated field experience. However, it was not possible to detect an improvement in PCK because their success in the classroom was determined by their mathematical knowledge.

Chick, Phan and Baker (2006) developed a framework for analysing PCK given in Table 1. This framework organises PCK into three sections. The first section "clearly PCK" in which the mathematical content and pedagogy are inseparable, includes teaching strategies, explanations, possible student misconceptions, and knowledge of resources and curriculum. The second section "content knowledge in a pedagogical context" includes the mathematics content as it is used in teaching. For example, being able to decompose the mathematical concepts into the main ideas and identify connections. The third section, "pedagogical knowledge in a content context" takes into account how generalized pedagogy is used in a mathematics classroom.

We will focus our analysis on the first section of this framework, clearly PCK: that is, we will analyse how the pre-service teachers respond in terms of what the student was thinking and how they as a teacher would respond.

Table 1
Framework for analysing Pedagogical Content Knowledge (Chick, Pham and Baker, 2006)

PCK Category	Evident when the teacher
Clearly PCK	
Teaching Strategies	Discusses or uses strategies or approaches for teaching a mathematical concept
Student Thinking	Discusses or addresses student ways of thinking about a concept or typical levels of understanding
Student Thinking - Misconceptions	Discusses or addresses student misconceptions about a concept
Explanations	Explains a topic, concept or procedure
Cognitive Demands of Task	Identifies aspects of the task that affect its complexity
Appropriate and Detailed Representations of Concepts	Describes or demonstrates ways to model or illustrate a concept (can include materials or diagrams)
Knowledge of Resources	Discusses/uses resources available to support teaching
Curriculum Knowledge	Discusses how topics fit into the curriculum
Purpose of Content Knowledge	Discusses reasons for content being included in the curriculum or how it might be used
Content Knowledge in a Pedagogical Context	
Profound Understanding of Fundamental Mathematics	Exhibits deep and thorough conceptual understanding of identified aspects of mathematics
Deconstructing Content to Key Components	Identifies critical mathematical components within a concept that are fundamental for understanding and applying that concept
Mathematical Structure and Connections	Makes connections between concepts and topics, including interdependence of concepts
Procedural Knowledge	Displays skills for solving mathematical problems (conceptual understanding need not be evident)
Methods of Solution	Demonstrates a method for solving a maths problem
Pedagogical Knowledge in a Content Context	
Goals for Learning	Describes a goal for students' learning (may or may not be related to specific mathematics content)
Getting and Maintaining Student Focus	Discusses strategies for engaging students
Classroom Techniques	Discusses generic classroom practices

Method

A group of Australian researchers led by Rosemary Callingham (Callingham et al, 2012) administered an Australia-wide survey to determine both primary and secondary pre-service teachers' general views of mathematics, mathematics learning and teaching, attitudes to mathematics, confidence with mathematics, mathematics content knowledge and PCK. The purpose of the survey was to collect evidence that would allow universities to monitor their Mathematics Education courses in terms of student outcomes so that changes could be made based on that evidence.

In this project we chose three PCK questions from their Australia-wide survey. For the purposes of this paper only one, shown in Figure 1 will be discussed. The question was given to the students in the primary mathematics courses in both the B.Ed. Primary and Graduate Diploma Primary prior to any teaching on the topic. The pre-service teachers' responses were then read and used to inform our teaching. After the lecture and tutorial (and hopefully the set reading) the pre-service teachers were asked the same question to determine if there was any change in their thinking. The pre-service teachers were also asked to reflect on their learning for the week. Research assistants collected and collated the data to match the before and after responses, thus ensuring the students remained anonymous.

There were two groups of students. One group was postgraduate students completing a one year Graduate Diploma Primary Education who studied one mathematics curriculum/pedagogy course. The second group was second year B.Ed. Primary students who were studying the first of their two curriculum/pedagogy courses in Early Years Mathematics. About 35 students in each group were given the questions, however only about half chose to respond to both the initial and final surveys. In both courses students are required to sit a mathematics content exam (to year 9 standard) and achieve at least 80% to pass the course. Consequently each group had weekly two-hour mathematics content workshops as well as their four hours of lectures and tutorials focusing on curriculum and pedagogy.

Results and Discussion

In week 5 of the semester, halfway through their course, both groups were given the first PCK question on fractions shown in Figure 1. Fractions are a notoriously difficult concept which many primary pre-service teachers struggle with. Both groups had previously had a content workshop on f ractions. Our results were analysed using the relevant aspects of the framework in Table 1. (Not all aspects can be considered from a written response.)

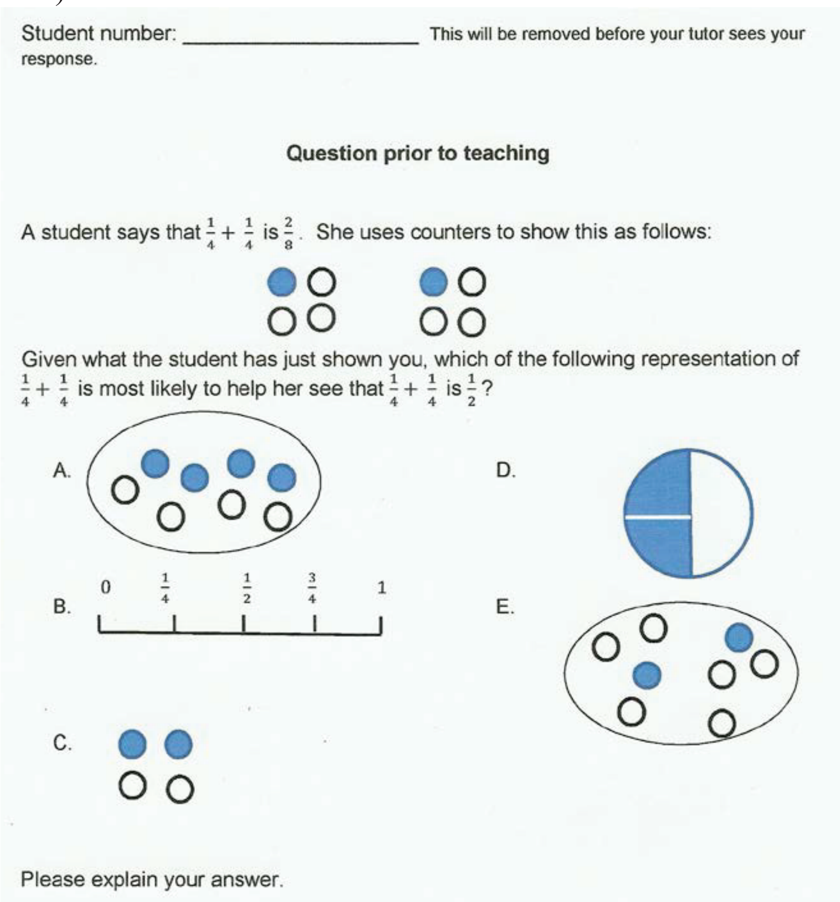


Figure 1. The question used to determine the pre-service teachers' PCK prior to and after teaching.

There has been much debate about the correct response to this question (CEMENT, 2012; Callingham et al, 2012) and we would suggest it is C though D could also be useful, particularly if it is used to confront the student's misconception and then one continues by referring to the set model used in C.

During the lecture, we discussed with the pre-service teachers: the different fraction models, (for example, region or area models, length models, and set models); ways to develop the concept, the language and the symbols; and possible sources of misconceptions and suggestions on how to avoid them. In the tutorial pre-service teachers were able to play with the different models and participate in a number of potential classroom activities. Whilst they were doing the activities, they were asked to think about the question; what would their future students be doing that would indicate that they had understood the concept? The pre-service teachers were also asked to consider the advantages and disadvantages of the different models in the different activities.

In the final activity each group was asked to draw on one of the whiteboards around the room how they would explain $\frac{1}{3} + \frac{1}{3}$. Earlier in the tutorial pre-service teachers had been asked to use the region, length and set model to demonstrate $\frac{1}{3} + \frac{1}{3}$ and discuss the strengths and weaknesses of each model. The students covered their boards in lots of different ways to show $\frac{1}{3} + \frac{1}{3}$ including: word stories for which they then drew 'the action'; different models; and different ways to work it out. Each board was very different and each group spoke to their board. Talking to the pre-service teachers as they were completing their activity revealed some of their misconceptions, for example, $\frac{1}{3} + \frac{1}{3} = \frac{2}{6}$ so that they could be addressed. This meant that in the final presentations there were no errors. Comments such as, "That's an easier way to do it than we have done" was common so the pre-service teachers were learning from the activity and potentially expanding their PCK.

We will present some of the pre-service teachers' responses using aspects of the framework in Table 1.

Teaching strategies and Explanations

The idea of the 'pie' or 'pizza' was the most common teaching strategy suggested. Most pre-service teachers chose D from Figure 1, the region model, as their response and this was due to the visual nature of the representation as they believed it was easier to explain as shown. They also found it difficult to put their explanations into words.

> Pre-service teacher 1F prior: "the two quarters can be seen separately but also collectively. it shows them as being added together and as one side is the same as the other it is ½

> Pre-service teacher 1F after: "cause you can physically see the ¼'s as well as when their added together they look like a ½ within the whole. the Pie represents the whole and the parts the best.

Teaching strategies

One pre-service teacher used the idea of layers with 4 hol es to represent the whole. Markers were then added to represent each quarter as shown.

> Pre-service teacher 2F: Use the idea of a layer and having four holes you give the student one layer with one marker + 3 empty spots then another layer with another ¼ showing that there are now $^{2}/_{4}$ which is half of the pieces.

Twenty of the 42 pre-service teachers who responded recognised that they needed to include the model that the student had used in the discussions.

> Pre-service teacher 3F: C because it is still using counters and if she adds the 1 dark circle with the other dark circle together she gets $^2/_4$ = ½

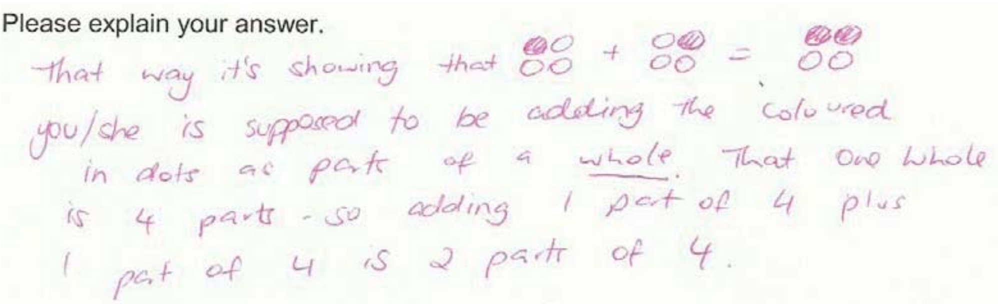
Also just because the pre-service teacher had recognised the need to discuss the same model that the student had used did not mean that the pre-service teacher had understood, for example, pre-service teacher 4F who chose A as their response.

> Pre-service teacher 4F prior: given the 1st diagram given A is most similarly in line. It shows more of a divide to establish the ¼ + ¼ principle than C would, as well as being able to show that there is ½ in total

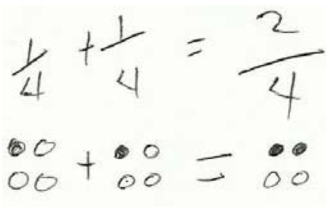
Explanations

Pre-service teachers found it very difficult to explain how the set model could be used to show $\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$. Only one pre-service teacher, 5F, referred to the whole being 4 counters. A couple of others just wrote it out without really explaining, for example 6F. Possibly the clearest attempt has been given by pre-service teacher 2F; however, they have not really referred to the whole being the 4 counters.

Pre-service teacher 5F:



Pre-service teacher 6F:



Student Thinking – misconceptions

Only two pre-service teachers, (out of the 42 responses) 7F and 8F, acknowledged the student's misconception. 7F acknowledged it only in terms of the equation (symbolic representation). Pre-service teacher 8F referred to the student's diagram and explained that the student has two wholes indicated and that when the fractions are added we want to arrange them on a single whole. That is, only one acknowledged the difficulty the student was having in interpreting the counters. The student was adding the 4 counters representing the ¼ to the 4 counters representing the other ¼ and had lost track of the fact that the whole was 4 counters and so when the two quarters were added there still needed to be 4 counters in the whole. This meant there should be 2 dark counters and 2 white counters.

> Pre-service teacher 7F: I would chose C because it seems that she is getting confused with adding the denominator as well by using C, it shows there is still 4 things instead of 8.

Pre-service teacher 8F after: because it is the same symbol that she has previously used. Just one whole instead of two.

Appropriate and Detailed Representations of Concepts

The student, in Figure 1, has been asked to show ¼ + ¼ in the question. She chose to use the set model to demonstrate her thinking. This would indicate her belief that it was the most appropriate. However, about half the pre-service teachers chose the region model, D, often referring to it as 'pizza' or 'pie'. This indicates where most of these people are most comfortable in their thinking. Therefore, they chose to show the student ¼ + ¼ = ½ using a region model and most spoke about the region model being the easiest one to understand.

When looking at the student's response to the question in Figure 1, these pre-service teachers did not change their method of helping the student before and after their university teaching.

One of the issues here is that the pre-service teachers have not thought about what the student in Figure 1 is thinking. This student is demonstrating the addition of fractions using the set model. The addition of fractions does not appear in the Australian curriculum until year 5. Hence the student is most likely in middle primary and has some understanding of fractions or she would not have chosen to use the set model to explain. Only one pre-service teacher, 8F, diagnosed the student's misconception by showing in her representation there were 2 wholes, that is, each quarter was ¼ of separate wholes. When the two ¼s were added they needed to be represented on the same whole. So whilst pre-service teachers are correct when they say that D, the region model will explain ¼ + ¼ = ½, this will not help the student in Figure 1 see that she has a difficulty with the set model and so has a misconception about the addition of fractions.

Conclusion

Although asking this question has given us an insight into the pre-service teachers' thinking, it has caused us to have a look at our own teaching. Many pre-service teachers have chosen D, the region model as they have considered it the most visual representation. To have a good PCK (Ball, 2000) one needs to have a deep mathematical understanding of the concept. We do not believe most of these pre-service teachers do have the necessary depth of content knowledge. These pre-service teachers were not able to diagnose the misconception or difficulty the student was having with the set model and so they were not able to provide appropriate feedback. Maybe they were not able to acknowledge the student's level of understanding because they are at the same place in their understanding. That is, most of these pre-service teachers do not understand the set model.

This then has implications for our teaching. Looking specifically at this example and how we addressed the items in Chick, Pham and Baker's (2006) framework within the lecture and tutorial, when discussing *Teaching Strategies,* we considered the region, length and set models of representing fractions, the language needed and then how to develop the symbols (*Appropriate and Detailed Representations of Concepts*). The region model was used initially to introduce the addition of fractions (but not with circles) and then with symbols by emphasizing 'that we can only add and subtract 'like terms'; an idea introduced earlier with the addition and subtraction of tens, ones etc'. We explored *Student Misconceptions* with decimals using some of the resources of Stacey and Steinle (Steinle, Stacey & Chambers (2006) and discussed what we could do to move students forward. Pre-service teachers explored *Appropriate and Detailed Representations of Concepts* and *Explanation* early in the tutorial and then again in the last activity of the tutorial.

However, we did not consider *Cognitive Demands of Tasks* or show *Profound Understanding of Fundamental Mathematics*. Perhaps our pre-service teachers would have benefitted if we had talked about the complexity of the decisions we make when deciding how to deal with students' thinking; how we decide the level of understanding we think the students have; and how we decide what we will do next. Next semester we are planning to identify specific mathematical concepts and work with these in more depth with our pre-service teachers, to share with them the *Cognitive Demands* of the specific the task and the *Profound Understanding of Fundamental Mathematics* involved in the task. We will also be showing our pre-service teachers more samples of primary students' work for them to contemplate why the students did what they did. This will be followed by discussion about the level of mathematical understanding of the primary students whose work we were examining and classroom activities to either address misconceptions or extend understanding.

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